Multivariate Control of a Phase Separation Process

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This paper presents a nonequilibrium mathematical model of a vapor-liquid separator. The model is used as the focus of a computer simulation control study conducted to provide the foundation for applying recent multivariate control developments to distillation systems.

Greenfield and Ward's structural analysis was found to provide reasonably good feedforward and decoupling control of pressure, liquid temperature, and liquid pressure. Because of implicit relationships between product composition variables and their manipulative inputs, structural analysis was inapplicable when these composition variables were among the controlled variables.

A method is presented which provides feedforward and some decoupling control for linear multivariable systems despite the existence of implicit relationships between controlled variables and their manipulative inputs. It is seen however that genuinely decoupled servo action on isolated state variables requires control through derivatives and thus an expansion of state space.

The effective control of a multivariable system is complicated by the dynamic intercoupling or interaction between the variables of the system. If this interaction can be removed, single variable feedback control theory can be independently applied to each control loop of the multivariable system. Early workers (*2, 6, 10*) used dynamic matrix operands to produce a one-to-one correspondence between the elements of the set point vector and the elements of the controlled variable vector of a linear multivariable system. Morgan (*14*) found that nondynamic controllers would give noninteracting servo control if all of the state variables of the system were monitored. Planchard (*15*) showed that both dynamic and nondynamic controllers derived from a linear system model could produce good quality noninteracting control when used on an experimental system which contained significant nonlinearities.

The efficacy of feedforward controllers in maintaining the controlled variables of a linear multivariable system invariate despite upsets in one or more of the system inputs has been demonstrated by Luyben and Lamb (*13*) and Tinkler (*17*). Bollinger and Lamb (*1*) advanced control technology a step further by combining feedforward and feedback control for multivariable systems. However, the feedback controllers were incapable of removing system intercoupling. More recently, Foster and Stevens (*5*) and Greenfield and Ward (*8, 9*) have presented methods which completely decouple linear multivariable systems while simultaneously providing for feedforward control of measurable input disturbances. Greenfield and Ward's powerful structural analysis (*8, 9*) represents the state of the art for the feedforward and decoupling control of systems for which such control is possible. However, as pointed out by Falb and Wolovich (*4*) and Gilbert (*7*), not all linear multivariable systems can be dynamically decoupled by state variable feedback.

To provide the groundwork for applying feedforward and dynamic decoupling control to distillation systems, this paper presents a control study (via digital computer simulation) of a vapor-liquid separator. A comprehensive nonequilibrium model which exhibits none of the numerical integration stability difficulties encountered by Distefano (*3*) is developed for the vapor-liquid separator and used as the focus for the control study. It will be shown that noninteracting control of the composition of the vapor and liquid phases of the separator is not possible by previously disclosed methods because of the existence of implicit relationships between these composition variables and the manipulative variables of the system. A control technique is presented which can produce feedforward and decoupling control for linear multivariable systems despite such implicit coupling. Computational difficulties precluded the complete demonstration of this methodology in the current simulation study, but the availability of system derivatives makes it experimentally feasible.

DEVELOPMENT OF MATHEMATICAL MODEL

The transient mathematical model developed for the nonequilibrium vapor-liquid separator shown in Figure 1 is based upon parameter lumping in the liquid and vapor phases and upon a film theory description of interphase mass and energy transport. The pertinent conservation and flux equations are

Mass Conservation:

Liquid phase

$$\frac{dl_j}{dt} = Fz_j - Lx_j - \Psi_{jl} A_i \quad (1)$$

for $j = 1, \text{---}, n$

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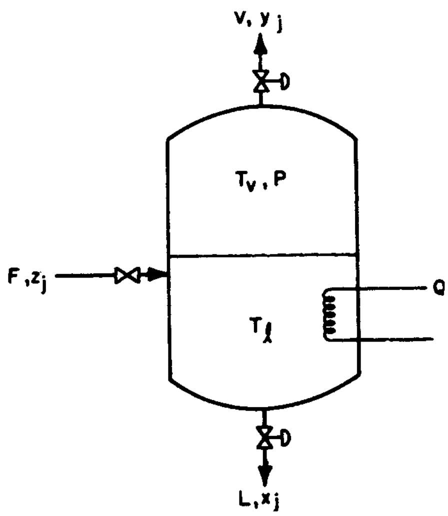

Fig. 1. Vapor-liquid separator.

Vapor phase

$$\frac{dv_j}{dt} = - Vy_j - \Psi_{jv} A_i \qquad (2)$$

$$\text{for} \quad j = 1, \text{- - -}, n$$

Energy Conservation:

Liquid phase

$$\frac{dE_l}{dt} = \sum_{j=1}^{n} (z_j H_{jf}) F + Q - q_l A_i$$

$$- \sum_{j=1}^{n} (x_j H_{jl}) L - \sum_{j=1}^{n} \Psi_{jl} A_i H_{jl}(T_i) \qquad (3)$$

Vapor phase

$$\frac{dE_v}{dt} = - q_v A_i - \sum_{j=1}^{n} \Psi_{jv} A_i H_{jv}(T_i) - \sum_{j=1}^{n} (y_j H_{jv}) V \qquad (4)$$

Momentum Conservation:

$$\text{FLOW} = \frac{C_0 S}{M_a} \sqrt{\frac{2g_c \rho (-\Delta P)}{1 - \beta^2}} \qquad (5a)$$

$$\beta = \frac{S}{S_t} \qquad (5b)$$

Film Theory Fluxes:

$$\Psi_{jl} = k_{jl} (f_l - f_i)_j \qquad (6)$$

$$q_l = h_l (T_l - T_i) \qquad (7)$$

$$\Psi_{jv} = k_{jv} (f_v - f_i)_j \qquad (8)$$

$$q_v = h_v (T_v - T_i) \qquad (9)$$

Interfacial Conservation Equations:

$$\Psi_{jl} + \Psi_{jv} = 0 \qquad (10)$$

$$\text{for} \quad j = 1, \text{- - -}, n$$

$$q_l + q_v + \sum_{j=1}^{n} \Psi_{jl} H_{jl} (T_i) + \sum_{j=1}^{n} \Psi_{jv} H_{jv} (T_i) = 0 \qquad (11)$$

The constitutive relationships between the component holdups and the liquid and vapor phase mole fractions are

$$x_j = \frac{l_j}{\sum_{j=1}^{n} l_j} \qquad (12)$$

$$y_j = \frac{v_j}{\sum_{j=1}^{n} v_j} \qquad (13)$$

Vapor and liquid phase fugacities were assumed to be given by the ideal solution forms:

$$f_{lj} = P_j^* x_j \qquad (14)$$

$$f_{vj} = P y_j \qquad (15)$$

Vapor pressures were calculated via the Antoine form and enthalpy was assumed to depend linearly on temperature throughout the present study. Vapor phase density is calculated from the ideal gas law, while liquid density is assumed to be a function of composition only. Within the context of these assumptions, a fourth-order Runge-Kutta numerical integration algorithm was used by Kleinpeter (*11*) to solve the above equations for a binary system composed of propane and normal butane. Table 1 lists the process parameters used in the computer solution of the equations. The absence of numerical integration stability problems [cf. Distefano (*3*)] in the model is attributed to not assuming equilibrium between the bulk liquid and vapor phases, including the fluid dynamic effects, and using vapor and liquid phase equations of state to ensure temperature and pressure consistency.

Before applying multivariable control techniques to this vapor-liquid separator, it was necessary to linearize the highly nonlinear model derived above. This was done by retaining only the first-order terms in the Taylor series expansion of each nonlinearity. The linearized equations are then written in terms of deviation variables (for example, $\overline{T} = T - T_{ss}$) and are Laplace transformed to give the following vector-matrix equation:

$$s \left[\begin{array}{c} Y^{(C)}{}_{C \times 1} \\ \hline Y^{(I)}{}_{I \times 1} \end{array} \right] = \left[\begin{array}{c|c} A^{(C,K)}{}_{C \times K} & A^{(C,M)}{}_{C \times M} \\ \hline A^{(I,K)}{}_{I \times K} & A^{(I,M)}{}_{I \times M} \end{array} \right] \left[\begin{array}{c} X^{(K)}{}_{K \times 1} \\ \hline X^{(M)}{}_{M \times 1} \end{array} \right] + \left[\begin{array}{c|c} B^{(C,C)}{}_{C \times C} & B^{(C,I)}{}_{C \times I} \\ \hline B^{(I,C)}{}_{I \times C} & B^{(I,I)}{}_{I \times I} \end{array} \right] \left[\begin{array}{c} Y^{(C)}{}_{C \times 1} \\ \hline Y^{(I)}{}_{I \times 1} \end{array} \right] \qquad (16)$$

This equation format assumes that all of the state variables are measurable and available for control purposes and that there are no system inputs whose values are unknown. (The matrix subscripts indicate their dimension.)

The state variables and system inputs of Equation (16) can be classified according to scheme of Bollinger and Lamb (*2*):

$Y^{(C)}{}_{C \times 1}$ — the $C \times 1$ vector of state variables which are to be controlled in a noninteracting fashion.

$Y^{(I)}{}_{I \times 1}$ — the vector of I state variables which are not controlled but which are measured and used to actuate control.

$X^{(K)}{}_{K \times 1}$ — the $K \times 1$ vector of measurable disturbance inputs.

$X^{(M)}{}_{M \times 1}$ — the $M \times 1$ vector of inputs which are manipulated to produce the desired control results.

Table 1. Process Parameters Used in Computer Simulation of Vapor-Liquid Separator

Component 1—Propane
Component 2—Normal butane

A_s	10 sq. ft.
C_o	0.61
h_{ss}	3.49 ft.
$h_l A_i$	4,000 B.t.u./(min.)(°F.)
$h_v A_i$	1,000 B.t.u./(min.)(°F.)
H_{1l}	36.8 $(T_l - 460)$ B.t.u./lb.-mole
H_{2l}	47.8 $(T_l - 460)$ B.t.u./lb.-mole
H_{1v}	8,520 + 16.6 $(T_v - 460)$ B.t.u./lb.-mole
H_{2v}	9,628 + 21.85 $(T_v - 460)$ B.t.u./lb.-mole
$K_{1l}A_i$	20 lb.-mole/(atm.)(min.)
$K_{2l}A_i$	5 lb.-mole/(atm.)(min.)
$K_{1v}A_i$	25 lb.-mole/(atm.)(min.)
$K_{2v}A_i$	6.25 lb.-mole/(atm.)(min.)
P_f	2 atm.
P_{ss}	1.48 atm.
P^*_1	exp $(10.23 - 4{,}259/T_l)$ atm.
P^*_2	exp $(10.98 - 5{,}370/T_l)$ atm.
Q_{ss}	1,667 B.t.u./min.
S_t	0.0873 sq. ft.
S_{fss}	9.75×10^{-3} sq. ft.
S_{lss}	1.998×10^{-3} sq. ft.
S_{vss}	8.10×10^{-4} sq. ft.
T_f	460.0 deg. R
T_{lss}	462.0 deg. R
T_{vss}	461.6 deg. R
V_s	100 cu. ft.
x_{1ss}	0.4383
y_{2ss}	0.1904
z_{1f}	0.50

The present treatment will first consider the case where

$$Y^{(C)}_{3\times 1} = \begin{bmatrix} \overline{T}_l \\ \overline{P} \\ \overline{h} \end{bmatrix} \;;\; Y^{(I)}_{3\times 1} = \begin{bmatrix} \overline{y}_2 \\ \overline{x}_1 \\ \overline{T}_v \end{bmatrix} \quad (17a, b)$$

$$X^{(K)}_{2\times 1} = \begin{bmatrix} \overline{T}_f \\ \overline{P}_f \end{bmatrix} \;;\; X^{(M)}_{3\times 1} = \begin{bmatrix} \overline{Q} \\ \overline{S}_v \\ \overline{S}_l \end{bmatrix} \quad (18a, b)$$

The coefficient matrices for the linearized model of the vapor-liquid separator model described earlier are

$$A^{(C,K)}_{3\times 2} = \begin{bmatrix} 4.61 \times 10^{-2} & -8.97 \times 10^{-2} \\ 0 & 3.54 \times 10^{-2} \\ 0 & 1.56 \times 10^{-1} \end{bmatrix} \quad (19)$$

$$A^{(C,M)}_{3\times 3} = \begin{bmatrix} 1.00 \times 10^{-3} & 0 & -9.54 \times 10^{-7} \\ 0 & -5.30 \times 10^{2} & -3.83 \times 10^{1} \\ 0 & 0 & -1.68 \times 10^{2} \end{bmatrix} \quad (20)$$

$$A^{(I,K)}_{3\times 2} = \begin{bmatrix} 0 & -6.35 \times 10^{-11} \\ 0 & 2.78 \times 10^{-3} \\ 0 & 4.54 \times 10^{-9} \end{bmatrix} \quad (21)$$

$$A^{(I,M)}_{3\times 3} = \begin{bmatrix} 0 & 0 & 6.86 \times 10^{-8} \\ 0 & 0 & 0 \\ 0 & -1.87 \times 10^{4} & -4.90 \times 10^{-6} \end{bmatrix} \quad (22)$$

$$B^{(C,C)}_{3\times 3} = \begin{bmatrix} -2.82 & 6.51 \times 10^{1} & -1.46 \times 10^{-11} \\ 1.90 & -4.51 \times 10^{1} & -4.72 \times 10^{-4} \\ -4.04 \times 10^{-2} & 1.05 & -2.08 \times 10^{-3} \end{bmatrix} \quad (23)$$

$$B^{(I,I)}_{3\times 3} = \begin{bmatrix} -8.00 \times 10^{1} & -1.97 \times 10^{2} & 8.02 \times 10^{-1} \\ 5.78 \times 10^{2} & 1.34 \times 10^{2} & 0 \\ -1.62 & -4.08 & 0 \end{bmatrix} \quad (24)$$

$$B^{(I,C)}_{3\times 3} = \begin{bmatrix} -1.21 \times 10^{-1} & 4.49 & 8.45 \times 10^{-13} \\ -6.10 \times 10^{-3} & 2.05 \times 10^{-1} & 3.64 \times 10^{-12} \\ 1.28 \times 10^{2} & 1.65 \times 10^{3} & -6.04 \times 10^{-11} \end{bmatrix} \quad (25)$$

$$B^{(I,I)}_{3\times 3} = \begin{bmatrix} -2.32 \times 10^{1} & -2.46 \times 10^{1} & 0 \\ -4.75 \times 10^{-1} & -8.09 \times 10^{-1} & 0 \\ -1.95 \times 10^{3} & -5.07 \times 10^{3} & -1.80 \times 10^{2} \end{bmatrix} \quad (26)$$

With the linear model thus specified, the structural analysis technique of Greenfield and Ward (*8, 9*) was implemented by moving the vector of manipulative inputs according to

$$X^{(M)}_{M\times 1} = F^{(M,M)}_{M\times M}\, X^{(M)}_{M\times 1} + F^{(M,K)}_{M\times K}\, X^{(K)}_{K\times 1} + F^{(M,I)}_{M\times I}\, Y^{(I)}_{I\times 1} + F^{(M,C)}_{M\times C}\, Y^{(C)}_{C\times 1} \quad (27)$$

The purpose of each of the above controller (F) matrices can be codified as follows:

$F^{(M,M)}_{M\times M}$ eliminates the forward intercoupling of the

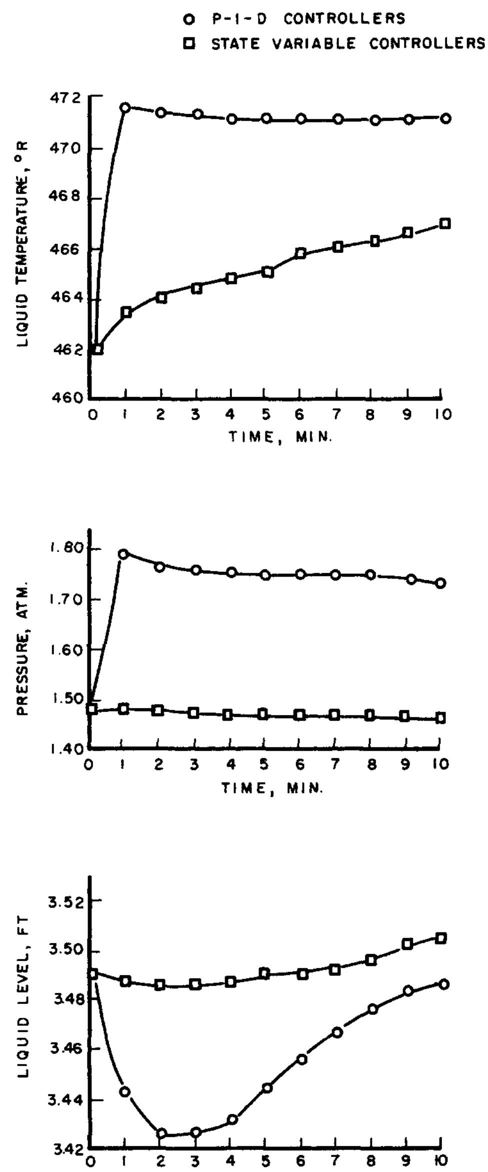

Fig. 2. Responses of controlled variables for a 9.24-deg. R step change in the set point of T_l.

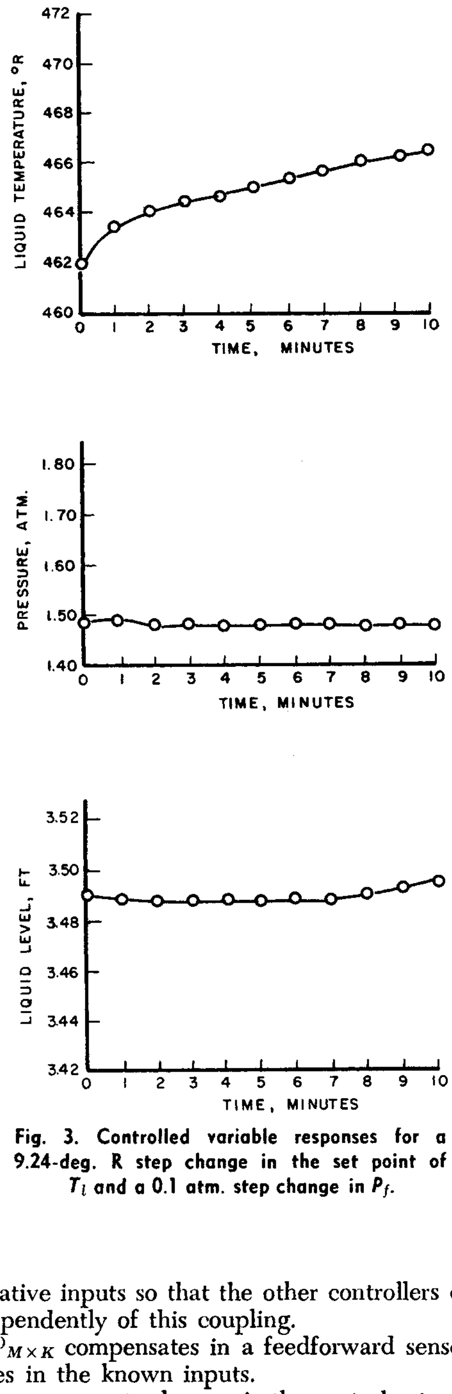

Fig. 3. Controlled variable responses for a 9.24-deg. R step change in the set point of T_l and a 0.1 atm. step change in P_f.

manipulative inputs so that the other controllers can operate independently of this coupling.

$F^{(M,K)}{}_{M\times K}$ compensates in a feedforward sense for disturbances in the known inputs.

$F^{(M,I)}{}_{M\times I}$ prevents changes in the control actuating state variables from effecting the controlled state variables.

$F^{(M,C)}{}_{M\times C}$ removes the intercoupling between the controlled state variables.

Because all of the state variables are measured and used for control purposes, each controller matrix contains only nondynamic elements which are easily calculated from the coefficient matrices of Equation (19) to (26) [via the relationships given by Greenfield and Ward (*8, 9*)].

To be useful, a control method based upon linear model dynamics must of course produce good results when applied to the nonlinear system from which the linear model was derived. The applicability of structural analysis for the control of the vapor-liquid separator was checked by simultaneously solving Equation (27) with the nonlinear equations of the system. In the digital computer solution (*11*), a Gauss-Seidel iteration was used to solve Equation (27) for $X^{(M)}{}_{M\times 1}$ and the integrations were performed with a fourth-order Runge-Kutta method. Upper and lower saturation limits were installed on the manipulative inputs so that Equation (27) could not force any manipulative input outside of the specified range. For the vapor and liquid valve flow areas, $\overline{S}_v$ and $\overline{S}_l$, the limits were such that the valves were either completely opened or closed or partially opened. Heat input was constrained to be between zero and three times the steady state value. The results are as described below.

CONTROL OF TEMPERATURE AND PRESSURE

Feed forward controller effectiveness was examined by simulating upsets in feed temperature and feed pressure. For a 9.20-deg.R. step change in feed temperature and 0.1 atm. step change in feed pressure, the linear model-based controllers were able to maintain the controlled variables of the nonlinear model essentially invariant. In these simulations perturbations in feed temperature and pressure affected the uncontrolled state variables, but the $F^{(M,I)}{}_{M\times I}$ controller prevented these changes from being transmitted to the controlled variables. To put these results in the proper perspective, a parallel study was carried out in which proportional-integral-derivative feedback controllers were used instead of the structural analysis controllers. Values of the gain, integral time, and derivative time were obtained by the Ziegler-Nichols reaction curve technique. It was found that the feedback controllers were almost as effective as the structural analysis controllers for regulator action alone.

The two control systems were also compared for servo control and it was here that the effect of variable interaction became evident. Figure 2 portrays the results of a 9.24-deg.F. step change in the set point of the liquid temperature. Although the proportional-integral-derivative controllers were able to quickly bring T_l to its new value, the other controlled variables were noticeably disturbed. The structural analysis controllers produced a reasonable degree on noninteraction. This ability to produce noninteraction is further demonstrated in Figure 3, which shows the system response for simultaneous set point and disturbance input changes.

In order to use structural analysis for servo control it was necessary to include an additional term in the controller definition of Equation (27). Because of the dynamic decoupling performed by the control law of Equation (27), a servo control term can be added as if each manipulative input-controlled variable pair is independent of other such pairs. Since the principal diagonal elements of $F^{(M,C)}{}_{M\times C}$ are all zero, one addend to Equation (27) that suggests itself would be

$$F^{D(M,C)}{}_{M\times C}\;(R^{(C)}{}_{C\times 1} - Y^{(C)}{}_{C\times 1})$$

However, when $F^{D(M,C)}{}_{M\times C}$ was chosen as the matrix of proportional controllers whose gains were calculated with the Ziegler-Nichols reaction curve technique applied to the nonlinear system decoupled by Equation (27), significant interaction occurred. The control action that led to the good results of Figures 2 and 3 was rather

$$F^{D(M,C)}{}_{M\times C}\;R^{(C)}{}_{C\times 1}$$

For complete system linearity and the absence of constraints on the manipulative inputs, both of the above terms would give perfect servo noninteraction. In the case at hand, the latter term has been shown to be less sensitive to the problems caused by nonlinearities and manipulative input constraints.

CONTROL OF PRODUCT COMPOSITION

Although temperature and pressure are variables commonly controlled in phase separation processes, the real control objective is to produce as good a separation of the most volatile and least volatile components as possible. Consider changing the control objective for the vapor-liquid separator so that

$$Y^{(C)}{}_{3\times 1} = \begin{bmatrix} \overline{x}_1 \\ \overline{y}_2 \\ \overline{h} \end{bmatrix}; \quad Y^{(I)}{}_{3\times 1} = \begin{bmatrix} \overline{P} \\ \overline{T}_l \\ \overline{T}_v \end{bmatrix} \qquad (28a, b)$$

The vectors $X^{(K)}{}_{3\times 1}$ and $X^{(M)}{}_{3\times 1}$ are the same as before [see Equations (18*a*, *b*)] and the *A* and *B* matrices of Equation (16) are rearranged consistent with the above changes. With the controlled state variables as specified in Equation (28*a*), the matrix $A^{(C,M)}{}_{3\times 3}$ is

$$A^{(C,M)}{}_{3\times 3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 6.90 \times 10^{-8} \\ 0 & 0 & -1.68 \times 10^{2} \end{bmatrix} \qquad (29)$$

Because of the singularity of this matrix, structural analysis cannot be used. While a terminal approach might be used, Kleinpeter (*11*) has shown that the resulting feedforward and decoupling controllers require high-order differentiation and are thus unsuitable.

The matrix of Equation (29) is singular because of what the authors term implicit coupling between the controlled state variables $\overline{x}_1$ and $\overline{y}_2$ and the corresponding manipulative inputs $\overline{Q}$ and $\overline{S}_v$. This implicit coupling is best explained with reference to Figures 4 and 5. Figure 4 is a signal flow graph of Equation (16) with *I*, *C*, and *M* each equal to one and *K* equal to zero. In signal flow graph representation, each node is a summing junction whose value is equal to the sum of the contributions of all incoming branches. The contribution of each branch is equal to the product of the branch transfer function and the value of the originating node. Figure 5 shows the case where $A^{(C,M)}{}_{1\times 1}$ is zero. Although the direct channel of information between $X^{(M)}{}_{1\times 1}$ and $Y^{(C)}{}_{1\times 1}$ has been removed, there is an indirect channel which is represented by the dotted lines of Figure 5. Such a relationship between a controlled variable and its manipulative input is what is meant by the terminology implicit coupling. In the vapor-liquid separator problem, $\overline{x}_1$ and $\overline{Q}$ are implicitly coupled because changes in $\overline{Q}$ cause variations in $\overline{x}_1$ by first producing changes in $\overline{T}_l$. A similar relationship exists among $\overline{y}_2$, $\overline{S}_v$, and $\overline{P}$.

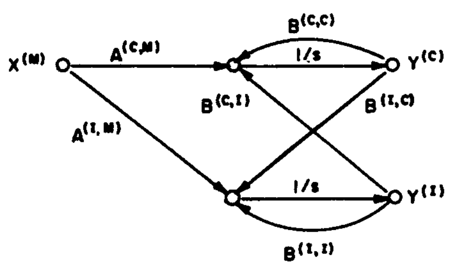

Fig. 4. Signal flow graph for a system in which implicit coupling is absent.

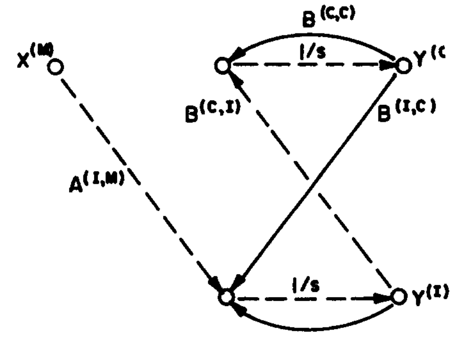

Fig. 5. Signal flow graph for an implicitly coupled system.

A method which would allow feedforward and decoupling control in spite of implicit coupling was then sought by considering the time domain version of Equation (16). If the bottom line of Equation (16) is inverted to the time domain and then solved for $Y^{(I)}{}_{1\times 1}$, the result is

$$Y^{(I)}{}_{I\times 1} = [B^{(I,I)}{}_{I\times I}]^{-1}\,[\dot{Y}^{(I)}{}_{I\times 1} - A^{(I,K)}{}_{I\times K}\,X^{(K)}{}_{K\times 1} - A^{(I,M)}{}_{I\times M}\,X^{(M)}{}_{M\times 1} - B^{(C,I)}{}_{C\times I}\,Y^{(C)}{}_{C\times 1}] \qquad (30)$$

Equation (30) is then substituted into the time domain analog of the top line of Equation (16) to yield

$$Y^{(C)}{}_{C\times 1} = A'^{(C,K)}{}_{C\times K}\,X^{(K)}{}_{K\times 1} + A'^{(C,M)}{}_{C\times M}\,X^{(M)}{}_{M\times 1} + B'^{(C,I)}{}_{C\times I}\,Y^{(I)}{}_{I\times 1} + B'^{(C,C)}{}_{C\times C}\,Y^{(C)}{}_{C\times 1} \qquad (31)$$

The primed matrices are related to the coefficient matrices of Equation (16) by

$$A'^{(C,K)}{}_{C\times K} = A^{(C,K)}{}_{C\times K} - B^{(C,I)}{}_{C\times I}\,[B^{(I,I)}{}_{I\times I}]^{-1}\,A^{(I,K)}{}_{I\times K} \qquad (32)$$

$$A'^{(C,M)}{}_{C\times M} = A^{(C,M)}{}_{C\times M} - B^{(C,I)}{}_{C\times I}\,[B^{(I,I)}{}_{I\times I}]^{-1}\,A^{(I,M)}{}_{I\times M} \qquad (33)$$

$$B'^{(C,I)}{}_{C\times I} = B^{(C,I)}{}_{C\times I}\,[B^{(I,I)}{}_{I\times I}]^{-1} \qquad (34)$$

$$B'^{(C,C)}{}_{C\times C} = B^{(C,C)}{}_{C\times C} - B^{(C,I)}{}_{C\times I}\,[B^{(I,I)}{}_{I\times I}]^{-1}\,B^{(I,C)}{}_{I\times C} \qquad (35)$$

Feedforward and dynamic decoupling control of a linear multivariable system would then result if the vector of manipulative inputs were moved according to

$$X^{(M)}{}_{M\times 1} = C^{(M,C)}{}_{M\times C}\,R^{(C)}{}_{C\times 1} + C^{(M,K)}{}_{M\times K}\,X^{(K)}{}_{K\times 1} + H^{(M,I)}{}_{M\times I}\,\dot{Y}^{(I)}{}_{I\times 1} + H^{(M,C)}{}_{M\times C}\,Y^{(C)}{}_{C\times 1} \qquad (36)$$

The nondynamic controllers which ensure the specified control objectives are

$$C^{(M,C)}{}_{M\times C} = [A'^{(C,M)}{}_{C\times M}]^{-1}\,D^{(1)}{}_{C\times M} \qquad (37)$$

$$C^{(M,K)}{}_{M\times K} = -\,[A'^{(C,M)}{}_{C\times M}]^{-1}\,A'^{(C,K)}{}_{C\times K} \qquad (38)$$

$$H^{(M,I)}{}_{M\times I} = -\,[A'^{(C,M)}{}_{C\times M}]^{-1}\,B'^{(C,I)}{}_{C\times I} \qquad (39)$$

$$H^{(M,C)}{}_{M\times C} = [A'^{(C,M)}{}_{C\times M}]^{-1}\,[D^{(2)}{}_{C\times M} - B'^{(C,C)}{}_{C\times C}] \qquad (40)$$

The diagonal matrices $D^{(1)}{}_{C\times M}$ and $D^{(2)}{}_{C\times M}$ are chosen so that the servo response will be rapid and the steady state offset will be small.

A linear approximation of $\dot{Y}^{(I)}{}_{I\times 1}$ for use in Equation (36) can be obtained by substituting Equation (36) into Equation (30) and then solving the resulting expression for $\dot{Y}^{(I)}{}_{I\times 1}$. The generation of $\dot{Y}^{(I)}{}_{I\times 1}$ in this fashion requires the inversion of the matrix

$$[I_{I\times I} - A^{(I,M)}{}_{I\times M}\,H^{(M,I)}{}_{M\times I}]$$

Upon computation, this matrix has been found to be singular (*11*). Rich (*16*) recently examined this difficulty by looking at a compact system of smaller dimension, namely

$$\dot{y}_1 = y_2 \qquad (41)$$

$$\dot{y}_2 = y_2 - x \qquad (42)$$

The parallel is to consider here the control of y_1 with x. Since

$$\dot{y}_1 = \dot{y}_2 + x \tag{43}$$

there is an understandable presumption that x can be used to obtain control over y_1 through this equation. Unfortunately, $\dot{y}_2$ and x are inextricably related through Equation (42) such that their sum is invariably y_2, a state variable established at any point in time by the history of the system and thus not subject to direct command. The bridge of x to y_1 must thus be sought through the derivative $\dot{y}_2$, for example, through

$$\ddot{y}_1 = \ddot{y}_2 = y_2 - x$$

The implementation of such a structured controller was not accomplished in the present work, but it clearly involves an expansion of the state space to embrace the derivative of the isolated variable (y_1 in the above case).

For exploratory purposes the term $H^{(M,I)}{}_{M \times I}\,\dot{Y}^{(I)}{}_{I \times 1}$ in Equation (36) was nulled (in effect reverting to conventional feedback without decoupling for part of the control vector). This partial decoupling scheme not unexpectedly gave good disturbance control, but the servo control quality was poor. Nevertheless, these results were significantly better than the results obtained using proportional-integral-derivative feedback controllers on each control loop. (As in the control of liquid temperature, pressure, and level, estimates of the optimum controller parameters were obtained by the Ziegler-Nichols reaction curve technique.)

Some reflection will suggest that the incidence of implicit coupling is not extraordinarily rare. In his recent work Rich (*16*) successfully implemented decoupled control through the second derivative as suggested above. His approach, a state variable method growing out of some ideas of Liu (*12*), is also less encumbered by linearity restrictions.

CONCLUSIONS

A comprehensive mathematical model of a vapor-liquid separator has been developed and used as the focus of a computer simulation control study aimed at providing the groundwork for the application of recent multivariable control techniques to distillation systems. None of the numerical integration stability problems experienced by Distefano (*3*) were encountered in solving the transient equations of the separator model. Other possible applications for this model are using it with experimental vapor-liquid separator data to generate film theory transfer coefficients, and employing the equations of the model as the single tray equations in a multistage distillation model.

When the controlled variables were liquid temperature, pressure, and liquid level, the structural analysis method of Greenfield and Ward (*8, 9*) provided reasonably good feedforward and decoupling control of the highly nonlinear system. Structural analysis was found to be inapplicable when the control variables were the mole fraction of the light component in the liquid product, the mole fraction of the heavy component in the vapor product, and liquid level because of the existence of implicit relationships between the composition variables and their corresponding manipulative inputs. While feedforward control and some measure of servo action were achieved here despite the existence of such implicit relationships, the need to expand the system dimension was demonstrated for accomplishing genuine decoupling control.

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NOTATION

A_i = area of vapor-liquid interface
A_s = cross-sectional area of vapor-liquid separator
C = number of controlled state variables
C_0 = valve coefficient
E = internal energy
f = fugacity
F = separator feed rate
g_c = Newton's law conversion constant
h = liquid level in vapor-liquid separator
h_l = liquid phase heat transfer coefficient
h_v = vapor phase heat transfer coefficient
H = enthalpy
I = number of control actuating state variables
k = mass transfer coefficient
K = number of measurable disturbance inputs
l_j = liquid phase molar holdup of component j
L = liquid flow rate
M = number of manipulative inputs
M_a = average molecular weight
n = number of components
P = system pressure
P° = vapor pressure
q = interphase heat flux
Q = external heat input
s = Laplace transform variable
S = flow area of a partially opened valve
S_t = flow area of totally opened valve
t = time
T = absolute temperature
v_j = vapor phase molar holdup for component j
V = vapor flow rate
V_s = volume of vapor-liquid separator
x = liquid phase mole fraction
y = vapor phase mole fraction
z = feed mole fraction
β = ratio S/S_t
Δ = difference operator
ρ = fluid density
Ψ = mass flux
Σ = summation operator

Matrices and Vectors

A = input coefficient matrix
B = state variable coefficient matrix
C = control matrix for implicitly coupled systems
D = diagonal matrix
F = structural analysis control matrix
H = control matrix for implicitly coupled systems
I = identity matrix
R = set point vector
X = vector of system inputs
Y = vector of state variables

Subscripts and Superscripts

C = controlled state variables
f = subscript indicating the feed stream
i = vapor-liquid interface subscript
I = designates control actuating state variables
j = subscript which indicates component j
K = designation for measurable system inputs
l = subscript denoting the liquid phase
M = identifies manipulative system inputs
s = subscript which refers to the vapor-liquid separator
ss = steady state value
v = subscript which denotes the vapor phase
. = differentiation with respect to time (as super-

script)
= deviation from the steady state value (as superscript)

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Velocity and Turbulence Measurements of Air Flow Through A Packed Bed

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Mean velocity and turbulence measurements in the void spaces of a cubic packing of equal spheres have been made at Reynolds numbers of 27,000, 10,000, 5,000 and 2,500, where N_{Re} is based on superficial air velocity and a sphere diameter of 7 cm. Two cubic arrangements were used: in the regular arrangement, the mean flow was parallel to one of the principal axes, while in the skewed arrangement, the mean flow made equal angles with the three principal axes of the packing.

Transverse mean velocity and turbulence intensity profiles across the center line of a central pore have been measured behind every bank and behind the bed for the regular arrangement of ten banks of spheres. The power spectrum and probability distribution of the fluctuating velocity have also been determined.

In a previous study (*1*) measurements of the pressure distribution on a sphere in a packed bed of uniform spheres in the cubic arrangement have shown that, generally speaking, the boundary-layer behavior on a sphere in a packing remains similar to that over a single sphere, when allowance is made for the effects of turbulence and of pressure gradient. The following qualitative description of the flow was deduced from the results: The laminar boundary layer on a sphere in the first bank separates at an angle ψ (measured from the front stagnation point) of about 90

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